Graduation
of
NPF & GSF Data

1 Introduction

1.1 This paper looks at the mortality experience for pensioners in the GSF and the NPF schemes and graduates this experience for use in the actuarial fields. Data were available for pensioners aged between 60 last birthday and 99 last birthday. Further details on the data are given in section 2.

1.2 The purpose of the graduation was to produce a fitted table that could be used by the NZ actuarial community. As the rates relate to retirement pensioners, the fitting of mortality rates below age 60 was not considered important in the graduation process. However, it was deemed necessary for rates to be available for ages 100 and above.

1.3 There have been a number of papers covering the mortality of very old people. These papers come to varying conclusions regarding the maximum human lifespan and to the exact shape of the mortality curve at those ages.

1.4 Given the range of differing opinions regarding the shape of the mortality curve for ages 100 and above (as well as the likely low financial significance at these ages), the initial idea was to graduate the data and extend the formula to the older ages.

1.5 Whilst this process resulted in acceptable rates for males, the female graduation did not result in rates that were regarded as acceptable. Some of the problems were decreasing rates at the youngest ages, rates being higher than the male rates, or rates decreasing at the extended ages (i.e., over 100).

1.6 The main reason for the problems at younger and older ages for the females was due to the relatively large variations in the crude mortality rates at each end of the age range. Some graduations were performed with added dummy rates at age 100 or with adjusted age 60 crude mortality rates.

1.7 The graduated rates with these dummy rates were markedly better although somewhat arbitrary. Where the adjustment was made at age 100, the lower ages were still less than satisfactory. Alternatively, adjusting age 60 either resulted in a poor fit or in the extended mortality rates not being acceptable.

1.8 Given the problems outlined above, it was decided to take the graduated rates for the given ages (i.e., 60 to 99 last birthday) and to blend these to 1 at age 120 using a blending formula similar to that adopted by the UK Actuarial Profession in its latest tables (the “00” series).

2 Data Supplied

2.1 The data supplied were in respect of pensioners from the NPF and the GSF for the three year period to 31 March 2005. The data related to pensioners aged 60 last birthday up to 99 last birthday.

2.2 In total there were 179,771 exposed to risk (being 106,296 for males and 73,475 for females) with a total of 6,953 deaths recorded (being 3,961 Males and 2,992 Females). Note that this is the total exposed to risk and deaths for the three year period ended 31 March 2005 (i.e., it is not an annualised number).

2.3 As a comparison, the latest Government Actuary data on the number of pensioners in registered superannuation schemes is summarised in the table below.

	Number Schemes	**Number Pensioners**	**% of Total**	**Schemes with: 100+**	**1,000+**
GSF/NPF	7	65,120	85.9%	5	4
Others	105	10,690	14.1%	35	0
Total	112	75,810	100.0%	40	4

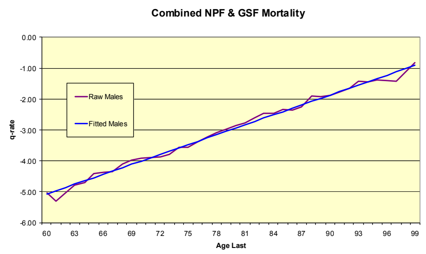
2.4 The exposed to risk and deaths for each age and sex is given in Appendix 1. Graphs of the raw q-rates and the logarithms of the q-rates are given in Appendix 2.

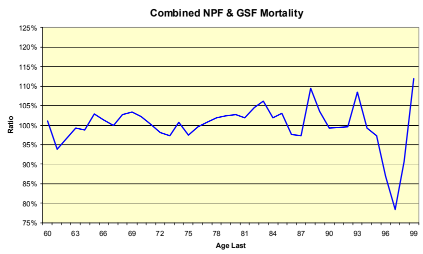
3 Graduation

3.1 A graduation has two components; namely goodness of fit and smoothness. There is a trade-off between these two components. The standard practice for graduating is to fit polynomials to the logarithms of either the μ_x or the q_x. This means that the actual underlying rates are exponential in character.

3.2 The most recent tables from the UK (the “00” series) are based on a graduation of the μ_x using the Gompertz-Makeham formula. For older ages, the rates were blended into an ultimate rate of 1 at age 120.

3.3 Initially, the raw data were graduated by fitting a polynomial to the logarithms of the q_x. For males, a good fit was found for a polynomial of order 1 (i.e., a straight line) as shown below by the graph of the fitted rates and the ratio of fitted to actual.





3.4 This was an extremely good fit with an r^2 of 0.992 and a F statistic of 4,666. However, this fit did not produce rates at older ages (from age 100) that looked reasonable. In particular, the q_x went to 1 at age 108 which was considered to be too young.

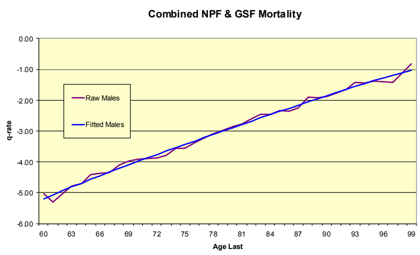
3.5 By increasing the order of the polynomial, the fit for ages 60-99 was not markedly improved (it already was a very good fit). However, the projected mortality was more reasonable. The fitted rates were produced using the following polynomial of order 4:

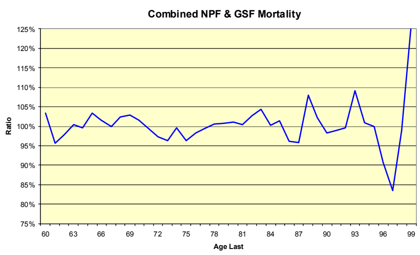
$$\ln(q_m) = a + bx + cx^2 + dx^3 + ex^4 \qquad (1)$$

where

a = -36.9188829832134
b = 1.35694617781617
c = -0.0231889862529791
d = 0.000192795632054175
e = -0.000000606263496572534

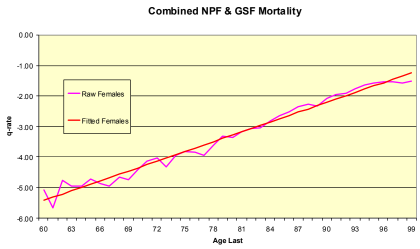
3.7 Again, this was an extremely good fit with an r^2 of 0.993 and a F statistic of 1,331. The corresponding fit and ratio of fit to actual are shown below.

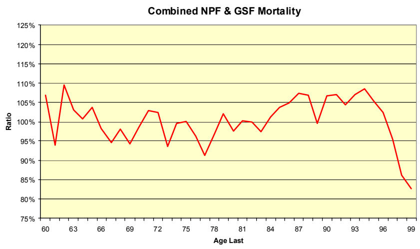




3.8 The extension to older ages is detailed in the next section.

3.9 For females, as for males, the polynomial of order 1 produced a good fit. This is shown below. This was an extremely good fit with an r^2 of 0.980 and a F statistic of 1,895.





3.10 Again, the extension to ages over 100 using the fitted formula was not considered reasonable. As with the graduation of the male data, higher order polynomials were considered. This did not result in an improvement in the projected mortality. The most likely reason for the poor fit was the very flat level of mortality observed in the raw data between ages 95 and 99.

3.11 It was decided to use the fit using a polynomial of order 1 as this gave the best results for the ages corresponding to the actual data. The fitted rates were produced using the following polynomial:

$$\ln(q_m) = a + bx \qquad (2)$$

where

a = -11.860667559317

b = 0.107246481187968

4 Extension to Older Ages

4.1 As the initial graduation was performed by fitting a single polynomial to the logarithms of the q_x, the easiest method of extending the rates to older ages would be by using the same formula. Whilst this gave reasonable results for males, the female rates projected on this basis were less reasonable.

4.2 The fitted logarithms were blended to 0 at age 120 (corresponding to a mortality rate of 1 at age 120). The use of age 120 as the upper age is not inconsistent with other published tables.

4.3 The blending from age 99 through 120 was by use of the formula:

$$\ln(q_x) = \frac{(120 - x)^{1.25}}{21} \times \ln(q_{99}) \qquad (3)$$

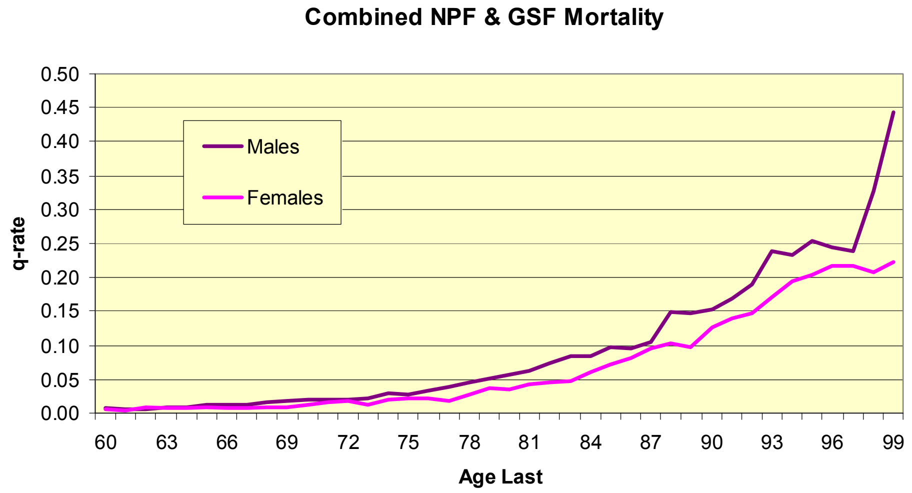
4.4 The final graduated rates are given in Appendix 3 with graphs of these rates in Appendix 4.

Appendix 1 - Data

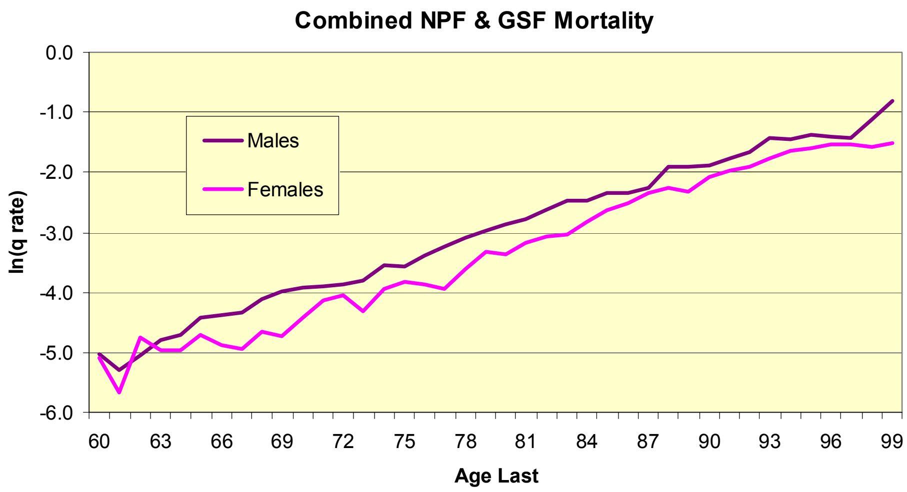
Age Last	Males			Females		
	Exposed	Deaths	q rates	Exposed	Deaths	q rates
60	2,889	19	0.0066	1,126	7	0.0062
61	3,506	18	0.0050	1,303	5	0.0035
62	3,892	25	0.0064	1,462	13	0.0086
63	3,984	33	0.0083	1,557	11	0.0071
64	3,890	35	0.0090	1,637	12	0.0070
65	3,912	47	0.0120	1,788	16	0.0089
66	3,952	50	0.0125	1,892	15	0.0077
67	4,017	53	0.0131	1,971	14	0.0071
68	4,145	68	0.0163	1,998	19	0.0095
69	4,305	80	0.0186	2,048	18	0.0088
70	4,502	89	0.0198	2,200	27	0.0120
71	4,654	95	0.0203	2,419	39	0.0161
72	4,709	99	0.0209	2,647	47	0.0176
73	4,709	106	0.0225	2,837	38	0.0134
74	4,684	134	0.0285	2,989	58	0.0194
75	4,614	131	0.0283	3,105	69	0.0221
76	4,531	154	0.0339	3,117	66	0.0212
77	4,400	173	0.0393	3,135	61	0.0193
78	4,211	191	0.0454	3,136	85	0.0269
79	3,920	200	0.0509	3,133	113	0.0361
80	3,581	205	0.0571	3,118	108	0.0346
81	3,306	205	0.0620	3,080	130	0.0420
82	2,981	219	0.0735	2,943	137	0.0464
83	2,494	211	0.0846	2,660	128	0.0479
84	2,038	173	0.0849	2,307	138	0.0598
85	1,690	164	0.0967	2,067	147	0.0709
86	1,434	136	0.0948	1,899	153	0.0806
87	1,211	127	0.1049	1,718	163	0.0949
88	1,029	153	0.1486	1,498	155	0.1035
89	804	119	0.1480	1,316	129	0.0977
90	633	96	0.1517	1,171	148	0.1260
91	475	81	0.1696	1,004	141	0.1404
92	359	68	0.1896	815	121	0.1478
93	273	66	0.2396	653	112	0.1715
94	197	46	0.2340	503	98	0.1940
95	138	35	0.2531	387	79	0.2041
96	92	23	0.2436	311	68	0.2169
97	67	16	0.2396	242	53	0.2173
98	44	15	0.3264	170	36	0.2083
99	25	11	0.4439	113	25	0.2218
All Ages	106,296	3,961		73,475	2,992	

Appendix 2 – Raw Mortality Rates

A2.1 Graph of q-rates



A2.2 Graph of ln(q-rates)

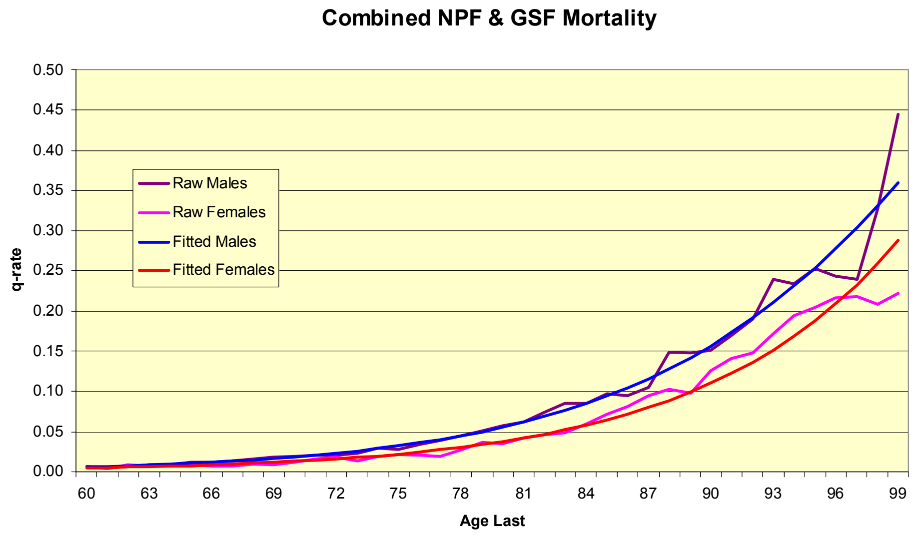


Appendix 3 – Fitted Rates

Age	Males	Females	Age	Males	Females
60	0.0055	0.0044	91	0.1731	0.1223
61	0.0063	0.0049	92	0.1909	0.1362
62	0.0072	0.0055	93	0.2102	0.1516
63	0.0081	0.0061	94	0.2311	0.1687
64	0.0092	0.0068	95	0.2536	0.1878
65	0.0104	0.0075	96	0.2777	0.2091
66	0.0117	0.0084	97	0.3033	0.2328
67	0.0132	0.0093	98	0.3305	0.2591
68	0.0148	0.0104	99	0.3590	0.2884
69	0.0166	0.0116	100	0.3814	0.3105
70	0.0186	0.0129	101	0.4049	0.3338
71	0.0208	0.0143	102	0.4296	0.3587
72	0.0232	0.0159	103	0.4554	0.3849
73	0.0259	0.0177	104	0.4823	0.4127
74	0.0290	0.0198	105	0.5103	0.4420
75	0.0323	0.0220	106	0.5395	0.4729
76	0.0360	0.0245	107	0.5698	0.5053
77	0.0401	0.0273	108	0.6011	0.5392
78	0.0447	0.0303	109	0.6335	0.5746
79	0.0497	0.0338	110	0.6668	0.6115
80	0.0554	0.0376	111	0.7010	0.6498
81	0.0616	0.0418	112	0.7359	0.6893
82	0.0685	0.0466	113	0.7715	0.7299
83	0.0761	0.0519	114	0.8073	0.7713
84	0.0846	0.0577	115	0.8433	0.8132
85	0.0939	0.0643	116	0.8790	0.8552
86	0.1042	0.0715	117	0.9140	0.8966
87	0.1155	0.0796	118	0.9472	0.9363
88	0.1280	0.0887	119	0.9775	0.9727
89	0.1417	0.0987	120	1.0000	1.0000
90	0.1567	0.1099			

Appendix 4 – Fitted Mortality Rates

A4.1 Graph of q-rates



A4.2 Graph of ln(q-rates)