Reinsurance and the distribution of term insurance claims

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Presented to the
NZ Society of Actuaries Conference

Queenstown - November 2006

1 Introduction

This paper investigates the effect of reinsurance on the distribution of net claims arising from a book of term insurance policies.

The paper is divided into several sections. Section 2 introduces the standard statistical model of claims behaviour, and gives expressions for the moments of the claims distribution. An “optimal reinsurance strategy” for a book of policies is defined as the set of retention levels (potentially different for each policy) that minimises a measure of the risk of excess net claims, for a given level of expected net cost. Algebraic expressions for optimal reinsurance strategies are derived for three risk measures. It appears that these results are not widely known, however the first one was derived, in a general insurance context, by de Finetti in 1940. Some implications of these results are considered.

Section 3 describes the use of Monte Carlo simulations to model claims experience. Practical issues facing this approach are discussed.

Section 4 gives the result of modelling the distribution of claims arising from a hypothetical book of term insurance. It confirms that the distribution of claims is positively skewed, the skewness decreasing as either the number of policies rises or the retention level falls. It is shown that the likely range of claims cannot be reliably estimated by assuming the claims distribution is Normal. APRA’s proposed “one year in 200” standard is used to estimate the level of capital required under in each scenario. A method for determining the optimum retention level, based on the marginal return on capital, is outlined.

Section 5 widens the stochastic model to include the occurrence of a pandemic. The possibility of a pandemic limits the diversification advantage an insurer expects to enjoy by increasing its scale. Many insurers will be unwilling to hold sufficient reserves to cope with all the claims that may occur in a severe pandemic, as they are unlikely to make an adequate return on this volume of capital.

An insurer is free to choose its own styles of reinsurance and retention levels. These decisions will be influenced by factors including the ease of administration, the level of reinsurance support required in underwriting and claims, existing reinsurance arrangements and company risk appetite. The type of analysis outlined in this paper gives an objective starting point for reinsurance decisions.

2 Minimum Risk Reinsurance Strategies

2.1 Statistical Model

Consider an insurer with *n* term life policies. Policy *k* has sum insured S_k, and probability q_k of claiming in the next period.

Let X_k be the random variable representing the total claims under policy *k* in the next period. Thus

$X_k = S_k$ with probability q_k,

$X_k = 0$ with probability $1 - q_k$.

Let $X = \sum_{k=1}^{n} X_k$, the total claims under the portfolio of policies in the next period. Clearly

$$\mu = E[X] = \sum_{k=1}^{n} S_k q_k \tag{1}$$

and, provided the claims under the policies occur independently, further statistics of the distribution of total claims are as follows:

$$\sigma^2 = E\left[(X-\mu)^2\right] = \sum_{k=1}^{n} S_k^2 q_k (1-q_k), \tag{2}$$

$$\mu_3 = E\left[(X-\mu)^3\right] = \sum_{k=1}^{n} S_k^3 q_k (1-q_k)(1-2q_k), \tag{3}$$

$$\gamma_2 = \sum_{k=1}^{n} S_k^4 q_k (1-q_k)(1-6q_k+6q_k^2).$$

μ_3, the third central moment, is also known as the third cumulant. γ_2 is the fourth cumulant, and from it the fourth central moment can be found as

$$\mu_4 = E\left[(X-\mu)^4\right] = \gamma_2 + 3\sigma^4.$$

From the cumulants the skewness and kurtosis of the claims distribution can be obtained:

$$SKEW[X] = {u_3}/{\sigma^3}$$

$$KURT[X] = {\gamma_2}/{\sigma^4}.$$

2.2 Reinsurance Strategy

The insurer now chooses that policy *k* has retention R_k, where $0 \le R_k \le S_k$. Let reinsurance strategy R be defined as $R = \{R_k : k = 1,...,n\}$, the set of retentions chosen for each policy. The reinsurer charges premium rate r_k for policy *k.* Hence the premium paid to the reinsurer is

$$\sum_{k=1}^{n} r_k \left(S_k - R_k\right)$$

and the expected net cost to the insurer (meaning expected loss of profit) due to this reinsurance arrangement is

$$\sum_{k=1}^{n} \left(r_k - q_k\right)\left(S_k - R_k\right).$$

Generally an insurer uses reinsurance to limit the fluctuation in annual claims experience, in particular to limit the severity of a "blow-out" in claims. An optimal reinsurance strategy is the reinsurance strategy that minimises a measure of the risk of excess net claims, across all the reinsurance strategies having the same expected net cost. The most common risk measure is the variance of the distribution of net claims.

2.3 Reinsurance strategy minimising variance

We now derive an expression for the reinsurance strategy that minimises the variance of net claims for a given level of expected net cost.

Let X_R be the random variable of the total claims net of reinsurance under reinsurance strategy R. We have

$$E\left[X_R\right] = \sum_{k=1}^{n} R_k q_k \tag{4}$$

$$Var\left[X_R\right] = \sum_{k=1}^{n} R_k^2 q_k \left(1 - q_k\right). \tag{5}$$

Let R be an optimal reinsurance strategy. Consider the effect of moving to reinsurance strategy $\hat{R} = \left\{\hat{R}_k : k = 1,...,n\right\}$ where

$$\hat{R}_i = R_i + \varepsilon$$

$$\hat{R}_j = R_j + \delta$$

$$\hat{R}_k = R_k \text{ for } k \neq i, j.$$

Let $X_{\hat{R}}$ be the random variable of the total claims net of reinsurance under reinsurance strategy $\hat{R}$. We see that

$$E\left[X_{\hat{R}}\right] = E\left[X_R\right] + \varepsilon q_i + \delta q_j.$$

The expected net cost of reinsurance strategy $\hat{R}$ is

$$\sum_{k=1}^{n} \left(r_k - q_k\right)\left(S_k - \hat{R}_k\right) = \sum_{k=1}^{n} \left(r_k - q_k\right)\left(S_k - R_k\right) - \varepsilon\left(r_i - q_i\right) - \delta\left(r_j - q_j\right).$$

Now we choose ε and δ so their combined effect on the expected net cost of the reinsurance is zero, ie

$$0 = \varepsilon\left(r_i - q_i\right) + \delta\left(r_j - q_j\right). \tag{6}$$

Moving to strategy $\hat{R}$ also affects the variance of the portfolio net of reinsurance, it becomes

$$Var\left[X_{\hat{R}}\right] = Var\left[X_R\right] + \left(2R_i\varepsilon + \varepsilon^2\right)q_i\left(1-q_i\right) + \left(2R_j\delta + \delta^2\right)q_j\left(1-q_j\right).$$

As strategy R was optimal, moving to $\hat{R}$ cannot reduce the variance, hence

$$0 \le \left(2R_i\varepsilon + \varepsilon^2\right)q_i\left(1-q_i\right) + \left(2R_j\delta + \delta^2\right)q_j\left(1-q_j\right),$$

independent of the choice of ε and δ.

This will only be true if its first order terms equal zero, ie

$$0 = R_i\varepsilon q_i\left(1-q_i\right) + R_j\delta q_j\left(1-q_j\right).$$

Substituting equation (6) into this gives

$$\frac{R_i q_i\left(1-q_i\right)}{r_i - q_i} = \frac{R_j q_j\left(1-q_j\right)}{r_j - q_j}.$$

As *i* and *j* were arbitrarily chosen, this expression must be true for all values of *i* and *j*, ie the expression

$$\frac{R_k q_k\left(1-q_k\right)}{r_k - q_k}$$

should be constant for all *k*. This means that

$$R_k = \frac{c\left(r_k - q_k\right)}{q_k\left(1-q_k\right)} \qquad (7)$$

for some constant *c*.

This result was first published by de Finetti in 1940. The author has not seen the original paper (it is in Italian!) but there are numerous discussions of it on the internet (search for “de Finetti” and “reinsurance”). The result is usually discussed in a general insurance context but applies equally to life insurance.

Consider that the reinsurance premium rate r_k can be expressed as

$$r_k = \left(1+\lambda_k\right)q_k$$

where λ_k represents a margin for the reinsurer’s expenses and profits. Substituting this into equation (7) shows that under an optimal reinsurance strategy

$$R_k = \frac{c\lambda_k}{1-q_k}$$

for some constant *c*. If the reinsurer’s margin λ_k is constant for all *k* then

$$R_k = \frac{d}{1-q_k}. \qquad (8)$$

for some constant *d*.

Under most policies the probability q_k of claiming in the next period is usually very small, so $\left(1-q_k\right)$ is very near to one. Making this approximation reveals that for the reinsurance strategy minimising variance the retention R_k must be constant for all *k*. This occurs under a Surplus Risk reinsurance treaty with a fixed maximum retention for each policy.

Note that each different retention level under a Surplus Risk treaty is an optimal reinsurance strategy for a different level of expected net cost. Nothing in the above analysis helps an insurer choose an appropriate retention level. Sections 4.5 – 4.7 of this paper looks at the implications of different retention levels on capital, and outline a method of selecting retention levels based on return on capital requirements.

It may happen that different groups of policies do have different reinsurer's margins. Typically this might occur for different product types, or between benefit types within a product. In this case the optimal retention for each product or benefit type is proportional to the reinsurer's margin for that product or benefit type. In other words if product group A has double the reinsurer's margin as product group B, the optimal retention for product group A would be double that of product group B.

The term $(1-q_k)$ in expression (8) is also worth examining. Consider two policies, each with $100,000 sum insured, the first with a 1% probability of claiming, the second with a 10% probability of claiming. Expression (8) shows that a slightly higher retention is preferred for the 10% policy than the 1% policy. One way of seeing this is to consider a portfolio of ten independent policies, each with $100,000 sum insured and 1% probability of claim. The expected total amount of claims under the ten 1% policies is $10,000, the same as under the single 10% policy. The single 10% policy could incur either $0 or $100,000 of claims. However under the ten 1% policies the total claims could be $0, $100,000, $200,000, or possibly up to $1,000,000. Clearly the group of ten policies is slightly riskier than the single 10% policy. Expression 3 allows for this by assigning the 1% policies a slightly lower retention than the 10% policy.

2.4 Reinsurance Strategy minimising μ_3 or γ_2

Variance is just one of a number of possible measures of risk. The main criticisms of variance as a risk measure are:

(a) That it penalises outcomes on either side of the mean, not recognising that low results are "good" and high results are "bad".

(b) Its weighting to extreme events may be inappropriate.

The advantages of variance are that it is widely understood, can be easily computed, and is often algebraically tractable.

Aggregate term insurance claims are always positively skewed, so there will be more very high results than very low results. Consequently the variance measure will place most weight on high outcomes, which gives some comfort that minimising variance should produce a sensible reinsurance strategy.

However it would be interesting to see the reinsurance strategy obtained by minimising a “downside-only” measure such as semi-variance.

The issue of the relative weighting placed on extreme results ultimately comes down to the risk appetite of the insurer. For example, consider a small insurer that will go broke if net claims are $5m over expected. In that case there is no point giving a higher risk weighting to the chance of $6m excess claims than to the chance of $5m excess claims! Potentially each insurer may have a different attitude to extreme events, select a different risk measure and consequently end with a different reinsurance strategy.

For most risk measures it is not possible to derive useful expressions for the resulting optimal reinsurance strategy. However, by adapting the method of section 2.3, expressions for the reinsurance strategies minimising μ_3, γ_2, or any of the higher cumulants can be found. The interested reader is invited to work through the details of this.

The μ_3 statistic does not suffer from criticism (a) of variance, as it recognises that outcomes below the mean are good (reduces the measure) and that outcomes above the mean are bad (increases the measure). The reinsurance strategy minimising μ_3 satisfies

$$R_k = c\sqrt{\frac{r_k - q_k}{q_k(1-q_k)(1-2q_k)}}$$

which is approximately equivalent to

$$R_k = c\left(1 + {}^{3q_k}\!/_{2}\right)\sqrt{\lambda_k} \ .$$

The fourth cumulant γ_2 contains some fourth order terms, so compared with variance it places a relatively higher weighting on the occurrence of extreme events. The reinsurance strategy minimising γ_2 satisfies

$$R_k = c\left(\frac{r_k - q_k}{q_k(1-q_k)(1-6q_k - 6q_k^2)}\right)^{1/3}$$

which is approximately equivalent to

$$R_k = c\left(1 + {}^{7q_k}\!/_{3}\right)\sqrt[3]{\lambda_k} \ .$$

Comparing these results to the equivalent expressions for the reinsurance strategy minimising variance shows:

- If the reinsurer’s margin is constant then the Surplus Risk reinsurance strategy is very close to optimal, independent of the choice of risk measure.
- Changes in reinsurer’s margin have less effect than they did when variance was minimised. If a product’s reinsurance margin was doubled the optimal retention would be 41% higher to mimimise μ_3 and 26% higher to minimise γ_2.

2.5 Summary

Surplus Risk reinsurance with a fixed retention is very close to the theoretically optimal reinsurance strategy, independent of what measure of risk is minimised. It may be that an insurance portfolio naturally subdivides into sub-portfolios, such as different benefit types, each with a different reinsurer's margin. In this case it is optimal for each sub-portfolio to have its own retention level, the retentions dependent on the relative level of the reinsurer's margin on that sub-portfolio.

3 Modelling Claims Experience

3.1 Simulation Process Outline

The occurrence of claims on a term insurance portfolio lends itself to analysis by computer simulation. This section outlines how claims can be modelled by a Monte Carlo method.

Consider a portfolio of term insurance on *n* lives insured. Let S_k be the sum insured on life *k* and q_k be the probability of a claim on life *k* in the next period.

Consider a reinsurance strategy with retention R_k on life *k*. For example under a Surplus Risk reinsurance treaty the retention is given by

$$R_k = \min(R, S_k)$$

where R is the maximum retention on any life insured.

Step 1 Let α_k be a random sampling from a Uniform [0,1] distribution. Thus Prob$\left[\alpha_k \leq z\right] = z$, for any $0 \leq z \leq 1$.

Step 2 Repeat Step 1 for each life insured *k*, ensuring all the α_k's are independent of each other. A claim occurs on life *k* if $\alpha_k < q_k$, and in that case the insurer's net claim cost is R_k.

Step 3 Determine the aggregate net claims cost the insurer faces under this sampling of the α_k. This is a single simulation of one year's claims experience of the term insurance portfolio.

Step 4 Repeat steps 1-3 to build up a picture of the distribution of aggregate term insurance claims. A sufficiently large number of simulations must be run to estimate the required parameters of the distribution. Depending on the nature of the parameters and the degree of accuracy required, the number of simulations needed may be from 500 to 10,000 or more.

3.2 Issues to consider

A number of practical issues should be considered before running a simulation.

Policy vs. Life Insured data. A key assumption in the statistical model is that claims under each policy occur independently. This is certainly not true when two policies are written on the same life insured! The obvious solution is to create a record for each life insured rather than for each policy. In practice this may not be entirely straightforward.

The first issue is identifying all instances where an individual is the insured under two or more policies. Depending on the available data this may be anywhere from simple to complex or unreliable. Once this is done the aggregate sum insured for each life insured can easily be found. More problematic may be determining a single representative q_k for the customer. Health, smoking and other assessments may legitimately vary between policies or within layers on a policy, and potentially date of birth and even sex may vary due to data entry errors. One approach is to set

$$q_k = \frac{\sum_i q_{k,i} S_{k,i}}{\sum_i S_{k,i}}$$

with the summations taken across all the different policies and layers on that life insured, and the $q_{k,i}$ being the best estimate of claim probability based on the details of that policy or layer.

Rider benefits. Many term insurance policies have rider benefits under which a different sum insured may be paid out. If required these can be handled by modifying the above algorithm. One way of doing this is as follows:

Let $q_{k,1}$ be the probability of a death claim on life *k*
Let $q_{k,2}$ be the probability of a TPD claim but not a death claim
Choose α_k at random in the normal way.
If $\alpha_k < q_{k,1}$ then death claim occurs
If $q_{k,1} \le \alpha_k < q_{k,1} + q_{k,2}$ then TPD claim occurs.

Lives with correlated risks. In any insurance portfolio there will be groups of lives whose probability of death is not independent. A simple example is a married couple who may both die in a car accident. Generally these effects are small enough to ignore. If it is necessary to model them it can be done in the following way.

Let customer 1 and 2 have probability of dying in the next period q_1 and q_2 respectively.
Let the probability of both dying in the next period be $q_{12} > q_1 q_2$.
Select α in the normal way.
If $\alpha < q_{12}$ then both die
If $q_{12} < \alpha < q_1$ then life 1 dies, life 2 survives
If $\alpha < q_1 + q_2 - q_{12}$ then life 2 dies, life 1 survives.

Software package. The necessary policy data will usually be obtained directly from the policy administration platform or from a valuation extract. This data could be imported into Excel, with one row for each policy. Excel has many advantages but does tend to "lock up" with large data sets, and is currently limited to 65,536 rows per sheet. Other database software, such as Access, may work better for large data sets.

4 Simulation results

4.1 Portfolio details

In this section the distribution of claims arising from a hypothetical portfolio of term insurance is considered. The base portfolio consists of 1,000 policies whose sums insured are given in the following table.

Sum Insured range	Number of policies
100 – 300k	250
300 – 600k	300
600k – 1m	250
1m – 2m	150
2m – 5m	50

The individual policies are evenly spaced within each sum insured range, eg $S_1 = 100{,}400$, $S_2 = 101{,}200$, …, $S_{999} = 4{,}910{,}000$, $S_{1000} = 4{,}970{,}000$.

The lives insured are assumed equally split between men and women, ages randomly chosen between 25 and 60, and mortality rates taken from table NZ01. Consequently the assumed probabilities of a claim on an individual life vary from 0.00031 to 0.0064.

On the base portfolio the expected number of claims was 1.57, the expected total claim amount was $1.23m, a standard deviation was $1.36m, and the distribution of claims had a skewness of 1.83. These values were computed directly from the policy data using formulae (1) – (3) of section 2.1.

When larger portfolios were required the base portfolio was replicated as many times as necessary.

4.2 Simulation Results

This following tables show the results of Monte Carlo simulations run on the portfolio of term insurance policies. In each case 10,000 simulations were run. Note that as the largest policy has a sum insured of $4.97m the line showing retention of $5m is in effect the gross portfolio.

Base portfolio – 1,000 policies

Retention Level	Mean Claims ($m)	Standard deviation ($m)	Skewness	95%-ile ($m)	99.5%-ile ($m)
$100k	0.16	0.12	0.75	0.40	0.60
$200k	0.31	0.24	0.76	0.80	1.00
$300k	0.43	0.35	0.79	1.09	1.50
$500k	0.64	0.53	0.86	1.61	2.37
$700k	0.79	0.67	0.94	2.08	3.01
$1m	0.93	0.83	1.04	2.50	3.72
$2m	1.12	1.10	1.27	3.26	5.02
$5m	1.22	1.33	1.78	3.99	6.61

5,000 policies

Retention Level	Mean Claims ($m)	Standard deviation ($m)	Skewness	95%-ile ($m)	99.5%-ile ($m)
$100k	0.79	0.28	0.31	1.30	1.60
$200k	1.53	0.54	0.32	2.48	3.10
$300k	2.18	0.78	0.35	3.53	4.43
$500k	3.21	1.19	0.38	5.29	6.62
$700k	3.93	1.51	0.41	6.58	8.31
$1m	4.63	1.85	0.44	7.92	10.1
$2m	5.60	2.48	0.55	10.1	13.2
$5m	6.13	3.05	0.81	11.8	16.1

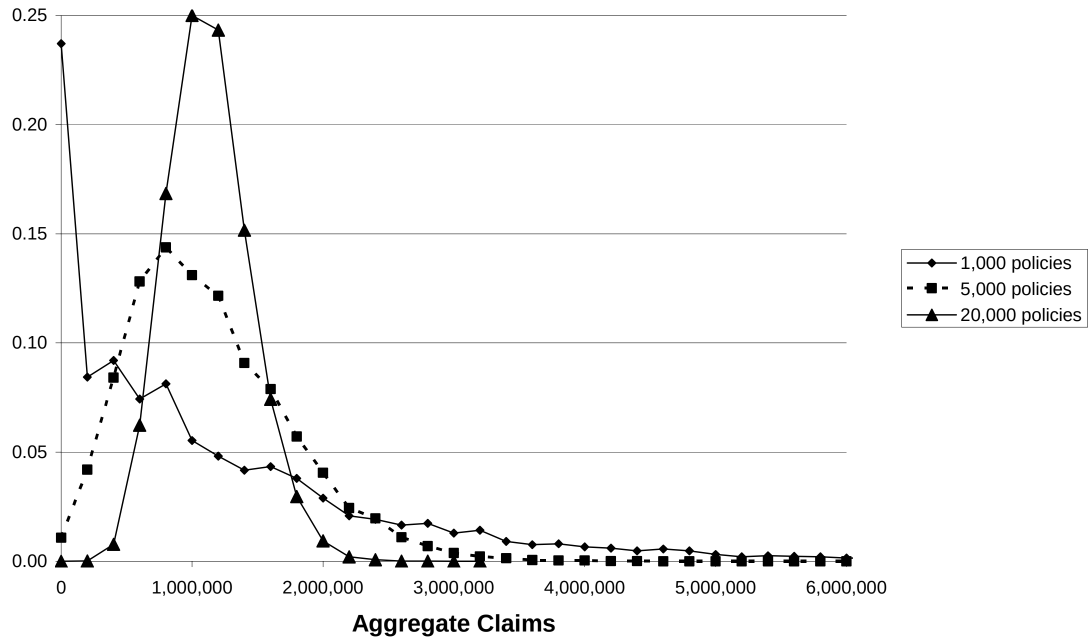
20,000 policies

Retention Level	Mean Claims ($m)	Standard deviation ($m)	Skewness	95%-ile ($m)	99.5%-ile ($m)
$100k	3.16	0.56	0.22	4.10	4.70
$200k	6.14	1.09	0.22	8.00	9.20
$300k	8.72	1.58	0.22	11.4	13.1
$500k	12.8	2.40	0.24	16.9	19.4
$700k	15.7	3.05	0.25	20.9	24.2
$1m	18.5	3.74	0.26	24.9	29.0
$2m	22.4	5.02	0.33	31.0	36.7
$5m	24.5	6.13	0.44	35.3	42.9

4.3 Observations from Simulation Results

Portfolio size. The portfolio's claim results become more stable as the portfolio size increases. The expected total claims increases in proportion to *n*, the number of policies. However the standard deviation increases in proportion to $\sqrt{n}$, and the skewness of the distribution is proportionate to $1/\sqrt{n}$. Thus as the number of policies rises the expected magnitude of any "blow-out" in claims falls relative to the level of expected claims. For instance the total claims expected once in every 20 years are 327% of expected claims for the portfolio of 1,000 policies, 192% for the portfolio of 5,000 policies, and 144% for the portfolio of 20,000 policies. This demonstrates the natural advantage an insurer enjoys by increasing its scale, through the pooling of independent risks.

The following graph shows the distributions of aggregate claims for the three portfolio sizes. The mean of the distributions have been scaled to be the same so the differing shape of the distributions can be seen on one graph.



This shows the progression from extremely long-tailed aggregate claims distribution for the portfolio of 1,000 policies, to the near Normal shape for the portfolio of 20,000. This is essentially a demonstration of the Central Limit Theorem.

Retention level. Reinsurance is effective in limiting the total variation of a portfolio's claim results. It is much less effective in reducing the magnitude of a claims "blow-out" relative to the level of expected claims. Consider the portfolio of 5,000 policies. The claims expected once every 20 years is 193% of expected claims for the gross portfolio, 171% if retention is $1m, and 165% if retention is $100k. Even if the retention was only $1 this ratio would not fall below 165%.

Accuracy of estimated parameters. Different parameters require very different numbers of simulations for estimation to the same degree of accuracy. For the portfolio of 5,000 policies, across all the retention levels, the estimate of mean claims is always accurate to within 0.15%. On the same portfolio the estimates of standard deviation are out by up to 1.4%, and skewness estimates by up to 13%. Broadly this is due to the relative sensitivity of the three measures to outliers. This can be seen by examining each policy's contribution to mean, variance and skewness per the summations in formulae (1) - (3). Of the 1,000 policies in the base portfolio, the top 117 contribute half of μ, the top 36 contribute half of σ^2, and the top 14 contribute half of μ_3.

4.4 Estimation of range of claims

The mean and variance of the net claims distribution can be directly calculated from the policy list using formulae (4) and (5). It is tempting to estimate a confidence interval for claims by assuming a Normal distribution. For example the 95% confidence interval is often stated as $\mu \pm 1.96\sigma$, and the 99% confidence interval as $\mu \pm 2.58\sigma$. However the simulation showed that the actual claims distribution is positively skewed, so the above ranges are likely to be inaccurate. In particular they are likely to underestimate the extent of a blow-out of claims. The following table gives the observed ranges that 95% and 99% of claims fell within. The tails are of equal size, for example the 95% confidence internal is bounded by the 2.5%-ile and the 97.5%-ile of the claims distribution. The ranges are expressed as number of standard deviations away from the mean.

Portfolio Size	Confidence level	Retention \$100k	Retention \$500k	Retention \$5m
1,000	95%	-1.27 to +1.96	-1.21 to +2.39	-0.91 to +2.72
	99%	-1.27 to +2.77	-1.21 to +3.26	-0.91 to +4.04
5,000	95%	-1.77 to +2.20	-1.80 to +2.18	-1.53 to +2.29
	99%	-2.13 to +2.93	-2.18 to +2.87	-1.78 to +3.27
20,000	95%	-1.89 to +2.03	-1.85 to +2.05	-1.74 to +2.17
	99%	-2.42 to +2.75	-2.32 to +2.74	-2.16 to +2.99

The actual claims ranges are most asymmetric for the smaller portfolio sizes and the highest retentions. This table demonstrates the necessity to run simulations to reliably estimate the likely range of claims.

4.5 Marginal Capital Requirement

Solvency and Capital Adequacy standards give detailed rules for determining the minimum capital an insurer should hold. Here we are specifically interested in the effect on the total capital requirements of changing the retention level on a book of term insurance. For the purposes of this paper, this marginal capital requirement is estimated by the "Capital for Claims Fluctuation" ("CFCF"). This is defined as

- 10% of the expected claims (net of reinsurance), plus
- the difference between the 99.5%-ile of the net claims distribution and the mean of the net claims distribution.

This definition, while very simplistic, takes into account the risk of claims being greater than expected due to mis-estimating the mean of the claims distribution, and of claims being greater than expected due to the random occurrence of individual claims. The 10% figure is based on GN5's requirement to use 110% of best estimate mortality for prudential reserving. The 99.5% figure was chosen due to APRA's proposal to include a "one year in 200" requirement in determining capital levels. It is acknowledged that this requirement is not currently included in GN5.

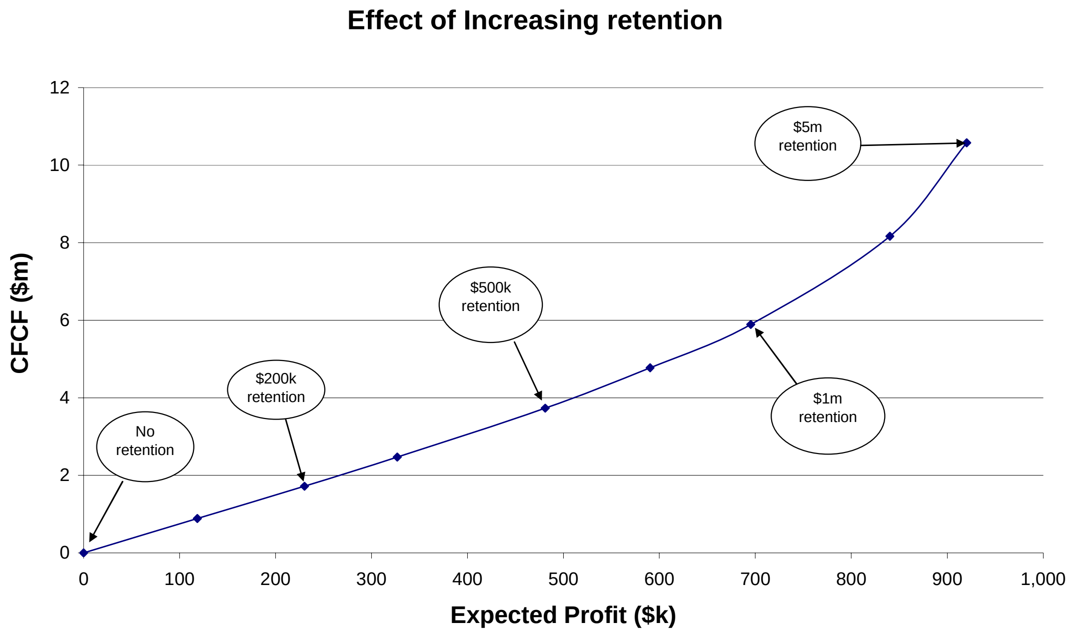
If an insurer totally reinsured their book the CFCF would be zero. As the retention level increases the CFCF will also grow. The following table shows the CFCFs required for the portfolios.

CFCF ($m)	Portfolio size		
Retention level	1,000 policies	5,000 policies	20,000 policies
$100k	0.4	0.9	1.9
$200k	0.7	1.7	3.7
$300k	1.1	2.5	5.3
$500k	1.8	3.7	7.9
$700k	2.3	4.8	10.0
$1m	2.9	5.9	12.3
$2m	4.0	8.2	16.6
$5m	5.5	10.6	20.8

This confirms that insurers are advantaged by increasing their number of independent risks, as the capital requirements vary roughly with the square root of the number of policies. This is because, for the above portfolios, the CFCF is dominated by the second term, being the allowance for the random occurrence of claims. The portfolio would have to contain approximately one million independent risks before the first term becomes larger than the second. From that point on the CFCF starts to increase more in proportion to the number of policies, rather than to its square root. In other words, once an insurer has taken on one million independent risks it has secured the majority of the advantage (in terms of CFCF) it can ever achieve by increasing its scale.

4.6 Return on capital and selecting retention level

By reinsuring its policies an insurer is giving away expected profit, in the form of the reinsurer's margin. Taking on more risk increases an insurer's net claims volatility and requires the insurer to hold additional CFCF. The following graph illustrates the trade-off between expected profit and capital. Expected profit is calculated assuming a reinsurer's margin of 15% plus a 5% investment return on the CFCF. The graph is for the portfolio of 5,000 policies.



The graph can be regarded as a risk/return picture, as CFCF is effectively a risk measure. Note how the curve becomes increasingly steep as retention increases. This shows that increasingly large amounts of risk, and additional capital, need to be taken on to generate every extra dollar of expected profit. In other words the marginal return on capital falls as retention increases - a classic example of diminishing returns.

4.7 Optimal Retention Level

Most companies operate with a target return on capital. To maximise an insurer's return on capital, retention should be set at the level where the marginal return on capital equals the target return on capital. The following table shows the marginal return on CFCF for the insurance portfolios considered above.

Marginal return on CFCF	Portfolio size		
Retention level	1,000 policies	5,000 policies	20,000 policies
0 to $100k	11.6%	18.3%	30.5%
$100k to $200k	11.1%	18.4%	29.5%
$200k to $300k	10.0%	17.9%	28.9%
$300k to $500k	9.5%	17.2%	29.0%
$500k to $700k	9.4%	15.5%	25.3%
$700k to $1m	8.6%	14.4%	23.3%
$1m to $2m	7.5%	11.4%	18.6%
$2m to $5m	6.0%	8.3%	12.7%

Assume that the target return on capital is 15%. The insurer with 1,000 policies should fully reinsure the book, as the expected profit from retaining risk never provides an adequate return on the extra capital employed. The insurer with 5,000 policies should set a $700k retention for optimal capital usage. Similarly the insurer with 20,000 policies should set a $2m retention.

In practice an insurer's decision about retention levels will be influenced by many factors, including ease of administration, the level of reinsurance support required in underwriting and claims, existing reinsurance arrangements and company risk appetite. However the type of analysis outlined above gives an objective starting point for these decisions.

5 Modelling a Pandemic

5.1 Pandemic Scenarios

Pandemics of varying intensities have occurred for hundreds of years and it is likely they will continue to occur. From a claims modelling viewpoint a pandemic can be represented by a period in which the probability of claim for every risk in the portfolio is raised. In this sense pandemics represent an additional risk that can't be easily diversified away. In this section the possibility of a pandemic is included in the claims modelling.

The change to the underlying model is as follows. In any year there is a 97% chance that no pandemic occurs. There is a 2% chance that a moderate pandemic occurs and a 1% chance that a severe pandemic occurs. If a moderate pandemic occurs then the probability of claim on each policy is increased by 0.001. If a severe pandemic occurs than the probability of a claim on each policy is increased by 0.004.

In a year that a pandemic doesn't occur, on the base portfolio of 1,000 policies we expect 1.6 claims totalling $1.2m. If a moderate pandemic occurs we expect 2.6 claims totalling $2.01m. If a severe pandemic occurs we expect 5.6 claims totalling $4.36m. Over the long term the total expected claims only rises 2.2% by including the possibility of pandemics.

There is no implication that the parameters chosen are a realistic estimate of either the likelihood of a pandemic or the actual effects a pandemic may have on mortality. However the chances of a pandemic occurring are based on the occurrence of three influenza pandemics in the 20th century. The effect on mortality is within the ranges given for moderate and severe pandemics in Alex Stitt's excellent paper.

5.2 Simulation Results

Incorporating the possibility of pandemics is very difficult in an algebraic model of claims. In particular the resulting claims distributions are likely to be very different from any standard distribution. However it was simple to modify the Monte Carlo simulation to include pandemics. Again 10,000 simulations were run. The following tables show the results of simulations incorporating the pandemic scenarios.

Base portfolio – 1,000 policies

Retention Level	Mean Claims ($m)	Standard deviation ($m)	Skewness	95%-ile ($m)	99.5%-ile ($m)
$100k	0.16	0.13	1.14	0.40	0.60
$200k	0.32	0.26	1.13	0.80	1.20
$300k	0.45	0.37	1.13	1.17	1.80
$500k	0.66	0.57	1.16	1.68	2.77
$700k	0.82	0.72	1.20	2.10	3.49
$1m	0.96	0.88	1.26	2.60	4.25
$2m	1.16	1.16	1.41	3.38	5.53
$5m	1.26	1.39	1.87	4.15	6.94

5,000 policies

Retention Level	Mean Claims ($m)	Standard deviation ($m)	Skewness	95%-ile ($m)	99.5%-ile ($m)
$100k	0.82	0.35	2.20	1.30	2.80
$200k	1.59	0.68	2.18	2.60	5.50
$300k	2.26	0.98	2.14	3.72	7.80
$500k	3.33	1.48	2.03	5.55	11.4
$700k	4.08	1.85	1.92	6.93	14.1
$1m	4.81	2.24	1.80	8.32	16.4
$2m	5.81	2.92	1.65	10.8	19.4
$5m	6.37	3.52	1.68	12.6	21.5

20,000 policies

Retention Level	Mean Claims ($m)	Standard deviation ($m)	Skewness	95%-ile ($m)	99.5%-ile ($m)
$100k	3.29	1.05	5.00	4.30	11.3
$200k	6.38	2.04	4.98	8.44	22.1
$300k	9.07	2.91	4.94	12.0	31.3
$500k	13.4	4.35	4.82	17.8	46.2
$700k	16.4	5.40	4.70	22.1	56.8
$1m	19.3	6.46	4.53	26.2	66.9
$2m	23.3	8.18	4.16	32.9	81.6
$5m	25.6	9.47	3.83	37.5	90.9

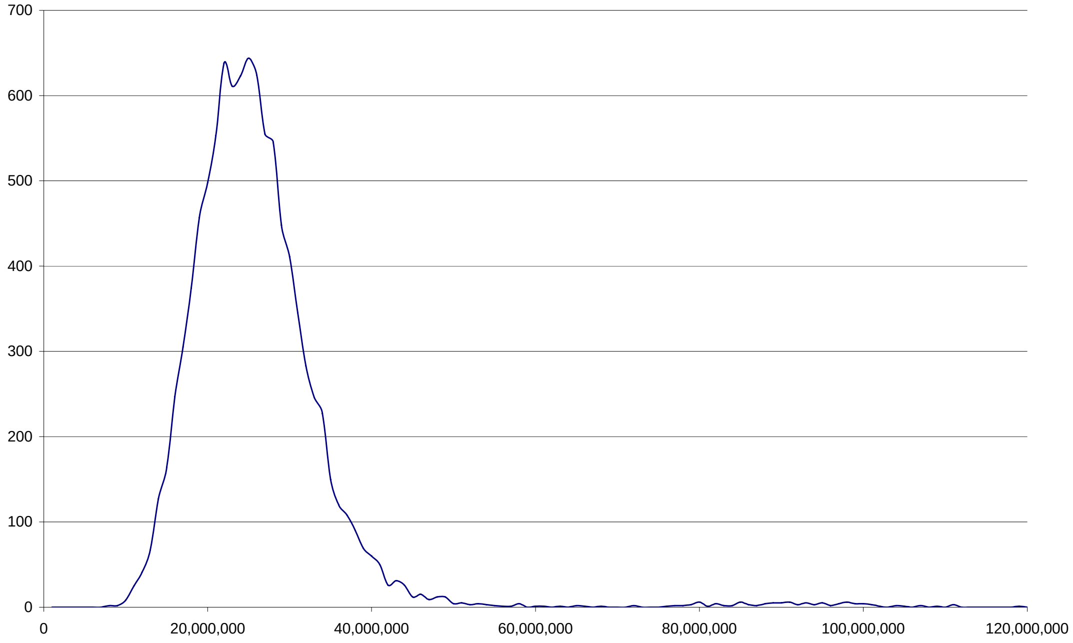
5.3 Observations from Simulation Results

Comparing with the results in section 4.2, we see that including the pandemic scenarios had very little effect on the claim distribution for the portfolio of 1,000 policies. Basically the portfolio is so small that claims were highly variable anyway, and the possibility of a pandemic hasn't made this much worse.

The claim distribution for the portfolio of 5,000 policies is more strongly affected by including pandemics in the modelling. Skewness is raised greatly, and ironically skewness now falls as retention increases. The 95%-iles are only slightly affected, increasing by around 5% from the non-pandemic levels. However the 99.5%-iles have increased by 50-100%. What is happening is that the distribution of claims is becoming tri-modal, one mode for each pandemic scenario.

This pattern is amplified for the portfolio of 20,000 policies. Standard deviation is at least 50% above its non-pandemic levels, and skewness has increased 10- to 20-fold. There is little change to the 95%-ile of the distribution, but the 99.5%-ile is more than double its pre-pandemic level.

The following graph shows the distribution of gross claims for the portfolio of 20,000 policies, based on the simulation results.



All of the occurrences of total claims over $80m were in years when a severe pandemic occurred, and it is these that have so greatly affected the summary statistics.

5.4 Estimation of range of claims

The following table gives the observed ranges that 95% and 99% of claims fell within. The ranges are expressed as number of standard deviations away from the mean.

Portfolio Size	Confidence level	Retention $100k	Retention $500k	Retention $5m
1,000	95%	-1.23 to +2.54	-1.18 to +2.36	-0.91 to +2.71
	99%	-1.23 to +3.29	-1.18 to +3.72	-0.91 to +4.07
5,000	95%	-1.48 to +1.94	-1.51 to +1.99	-1.39 to +2.28
	99%	-1.76 to +5.64	-1.82 to +5.48	-1.61 to +4.30
20,000	95%	-1.13 to +1.63	-1.14 to +1.63	-1.23 to +1.76
	99%	-1.32 to +7.63	-1.40 to +7.55	-1.50 to +6.90

Inclusion of the pandemic scenarios has made the 99%-iles even more asymmetric than before, particularly for the larger portfolios. In contrast the 95%-ile are tighter than before. This is primarily because the ranges are expressed as standard deviations away from the mean, and the standard deviations have grown significantly.

5.5 Marginal Capital Requirement

The following table shows the CFCF based on the portfolio.

CFCF ($m)	Portfolio size		
Retention level	1,000 policies	5,000 policies	20,000 policies
$100k	0.5	2.1	8.3
$200k	0.9	4.1	16.3
$300k	1.4	5.8	23.1
$500k	2.2	8.4	34.1
$700k	2.8	10.5	42.1
$1m	3.4	12.0	49.6
$2m	4.5	14.2	60.6
$5m	5.8	15.8	67.9

This shows that the insurer now gets only a small advantage by increasing the portfolio size. Moving from 5,000 to 20,000 policies the CFCF has increased fourfold.

5.6 Optimal Retention Level

The following table shows the marginal return on CFCF for the insurance portfolios considered above.

	Portfolio size		
Retention level	1000 policies	5,000 policies	20,000 policies
0 to $100k	10.4%	11.0%	10.9%
$100k to $200k	10.0%	10.8%	10.8%
$200k to $300k	9.2%	10.9%	10.9%
$300k to $500k	9.2%	11.0%	10.8%
$500k to $700k	8.9%	10.6%	10.7%
$700k to $1m	8.4%	12.0%	10.8%
$1m to $2m	7.7%	12.0%	10.5%
$2m to $5m	6.2%	10.2%	9.6%

The table shows that none of the insurers achieves the target return on capital by taking on insurance risk. This is a complete change from the no-pandemic scenario, and is caused by the need to keep enormous levels of capital to cope with a catastrophic event that may occur once a century.

The results above have to be balanced with practical considerations. Reinsurers won't generally be willing for an insurer to cede all risk, so a compromise position will have to be reached.

5.7 Discussion

Pandemics limit the ability of insurers to accept large numbers of risks and enjoy the benefits of diversification. By definition pandemics affect wide regions, but it is very likely that any pandemic will be more severe in some regions than others. This gives insurers, and in particular reinsurers, the ability to reduce their risk concentration by accepting risks across multiple countries and continents. Of course insurers achieve diversity by taking on different types of risks, such as property insurance.

The analysis above indicates that life insurers can't generally afford to hold sufficient capital to cope with all the claims that would occur under a particularly severe pandemic. To maintain a viable insurance industry the capital requirements for life insurers will have to be set at lower levels than those assumed in this paper. Ultimately this reinforces the point that each company's resources are finite, and that any insurer will fail to meet its obligations under the most extreme circumstances.